3 Exponential and Logarithmic Functions

3.4 Solving Exponential and Logarithmic Equations

What You Should Learn

- Solve simple exponential equations.
- Solve more complicated exponential equations.
- Use exponential equations to model and solve real-life problems.

Introduction

Introduction

Just read this slide and the next two. You do not need to write them down.

There are two basic strategies for solving exponential or logarithmic equations.

For $a > 0$ and $a \neq 1$ the following properties are true for all x and y for which

$$\log_a x \qquad \textit{and} \qquad \log_a y$$

are defined.

Introduction

One-to-One Properties

$$a^x = a^y \text{ if and only if } x = y.$$

$$\log_a x = \log_a y \text{ if and only if } x = y.$$

Inverse Properties

$$a^{\log_a x} = x$$

$$\log_a a^x = x$$

Example 1 – *Solving Simple Exponential and Logarithmic Exponential*

Original Equation	*Rewritten Equation*	*Solution*	*Property*
a. $2^x = 32$	$2^x = 2^5$	$x = 5$	One-to-One
b. $\log_4 x - \log_4 8 = 0$	$\log_4 x = \log_4 8$	$x = 8$	One-to-One
c. $\ln x - \ln 3 = 0$	$\ln x = \ln 3$	$x = 3$	One-to-One
d. $\left(\frac{1}{3}\right)^x = 9$	$3^{-x} = 3^2$	$x = -2$	One-to-One
e. $e^x = 7$	$\ln e^x = \ln 7$	$x = \ln 7$	Inverse
f. $\ln x = -3$	$e^{\ln x} = e^{-3}$	$x = e^{-3}$	Inverse
g. $\log_{10} x = -1$	$10^{\log_{10} x} = 10^{-1}$	$x = 10^{-1} = \frac{1}{10}$	Inverse
h. $\log_3 x = 4$	$3^{\log_3 x} = 3^4$	$x = 81$	Inverse

Introduction

The strategies used in Example 1 are summarized as follows.

Strategies for Solving Exponential and Logarithmic Equations

1. Rewrite the original equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions.
2. Rewrite an *exponential* equation in logarithmic form and apply the Inverse Property of logarithmic functions.

Applications

Example 12 – *Doubling an Investment*

You have deposited \$500 in an account that pays 6.75% interest, compounded continuously. How long will it take your money to double?

Solution:

Using the formula for continuous compounding, you can find that the balance in the account is

$$A = Pe^{rt}$$

$$= 500e^{0.0675t}.$$

To find the time required for the balance to double, let $A = 1000$, and solve the resulting equation for t.

Example 12 – *Solution*

cont'd

$500e^{0.0675t} = 1000$ Substitute 1000 for *A*.

$e^{0.0675t} = 2$ Divide each side by 500.

$\ln e^{0.0675t} = \ln 2$ Take natural log of each side.

$0.0675t = \ln 2$ Inverse Property

$$t = \frac{\ln 2}{0.0675}$$ Divide each side by 0.0675.

$t \approx 10.27$ Use a calculator.

The balance in the account will double after approximately 10.27 years.